



Multi-View Complementary Hash Tables for Nearest Neighbor Search

*Xianglong Liu**, *Lei Huang**, *Cheng Deng[†]*, *Jiwen Lu[‡]*, *Bo Lang**

**State Key Lab of Software Development Environment, Beihang University, Beijing, China*

[†]Xidian University, Xi'an, China

[‡]Tsinghua University, Beijing, China



Background and Related Work

Background

- The explosive growth of the vision data motivates the recent studies on hash based nearest neighbor search (NNS)
- Locality-Sensitive Hashing (LSH) is able to achieve compressed storage and efficient computation in NNS
- Building multiple hash tables and probing multiple buckets can boost the overall NNS performance

Related Work

- The most widely-used strategy: random LSH-based multi-table, working like multi-index hashing [27]
- Complementary hash tables: a sequential learning method [37]
- A general multi-table construction strategy: bit selection over existing hashing algorithms [21]

Main Issues

- It often requires a huge number of tables without eliminating the table redundancy
- Hash tables are usually learned only from single type of data source, while adaptively combining them can help learn more informative hash functions

Multi-View Hash Table

Notations

- A set of N training examples with M views
- $\mathbf{x}_n^{(m)} \in R^{d_m \times 1}$: the m -th feature (d_m dimension) of n -th sample
- $\mathbf{X}^{(m)} = [\mathbf{x}_1^{(m)}, \mathbf{x}_2^{(m)}, \dots, \mathbf{x}_n^{(m)}]$: the m -th feature of all data

Goal

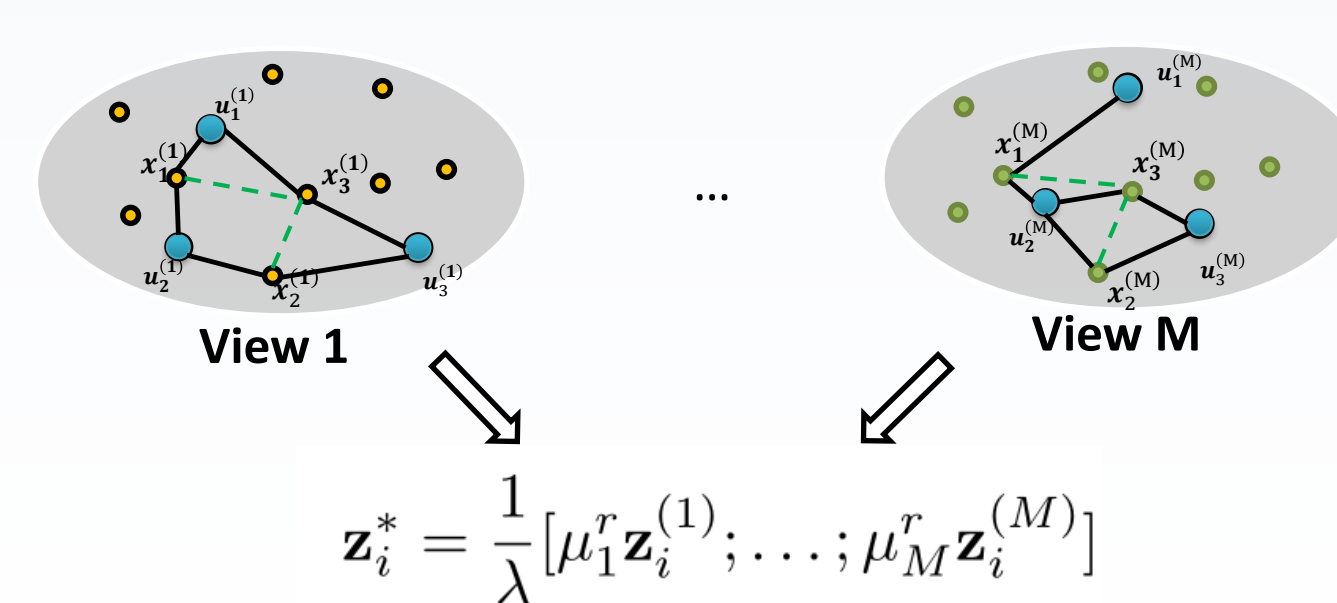
- Build L multi-view complementary hash tables $\{T_l\}_{l=1}^L$, with B hash functions $H = \{h_j(\cdot)\}_{j=1}^B$, $h_j(\cdot): R^d \rightarrow \{-1, 1\}$, learnt for each table.

Exemplars based Feature Fusion

- nonlinear feature mapping

$$[\mathbf{z}_i^{(m)}]_k = \frac{\delta_k^{(m)} \mathcal{K}(\mathbf{x}_i^{(m)}, \mathbf{u}_k^{(m)})}{\sum_{k'=1}^K \delta_k^{(m)} \mathcal{K}(\mathbf{x}_i^{(m)}, \mathbf{u}_{k'}^{(m)})}$$

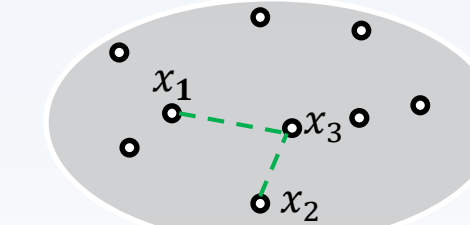
- feature fusion



• Data points • Data points in different views • Exemplars
- Similarity to nearest neighbors - Similarity to nearest exemplars

Hash Function Learning

- low-rank similarity



$$\mathbf{S} = \mathbf{Z}^* \mathbf{\Lambda}^{-1} \mathbf{Z}^{*T}$$

- alternating optimization

$$\begin{aligned} \max_{\mathbf{Y}} \quad & \text{tr}(\mathbf{Y}^T \mathbf{S} \mathbf{Y}) \\ \text{s.t.} \quad & \mathbf{1}^T \mathbf{Y} = 0, \mathbf{Y}^T \mathbf{Y} = N \mathbf{I}_{B \times B} \end{aligned}$$

step 1.1: $\mathbf{Y}$ -update $\mathbf{Y} = \mathbf{Z}^* \mathbf{W}$, $\mathbf{w}_b = \sqrt{N \sigma_b} \mathbf{\Lambda}^{-\frac{1}{2}} \mathbf{v}_b$
step 1.2: μ -update
step 2: $\mathbf{Y}^*$ -update $\mathbf{Y}^* = \text{sgn}(\mathbf{Z}^* \mathbf{W}^*)$, $\mathbf{W}^* = \mathbf{W} \mathbf{R}$

- out-sample-extension

Theorem 1: $\mathbf{z}(\mathbf{x}) = [\mu_1^T \mathbf{z}^{(1)}(\mathbf{x}), \dots, \mu_M^T \mathbf{z}^{(M)}(\mathbf{x})]^T$

$$\mathbf{y}^* = \text{sgn}(\mathbf{W}^{*T} \mathbf{z}(\mathbf{x}))$$

Complementary Multi-View Tables

Table Complementarity

- sequential learning: for each view the similarities on the inconsistent neighbor pairs will be amplified at next round

$$\hat{\mathbf{S}}_{ij}^{(m)} = \hat{\mathbf{S}}_{ij}^{(m)} \exp(-\alpha^{(m)} \mathbf{P}_{ij})$$

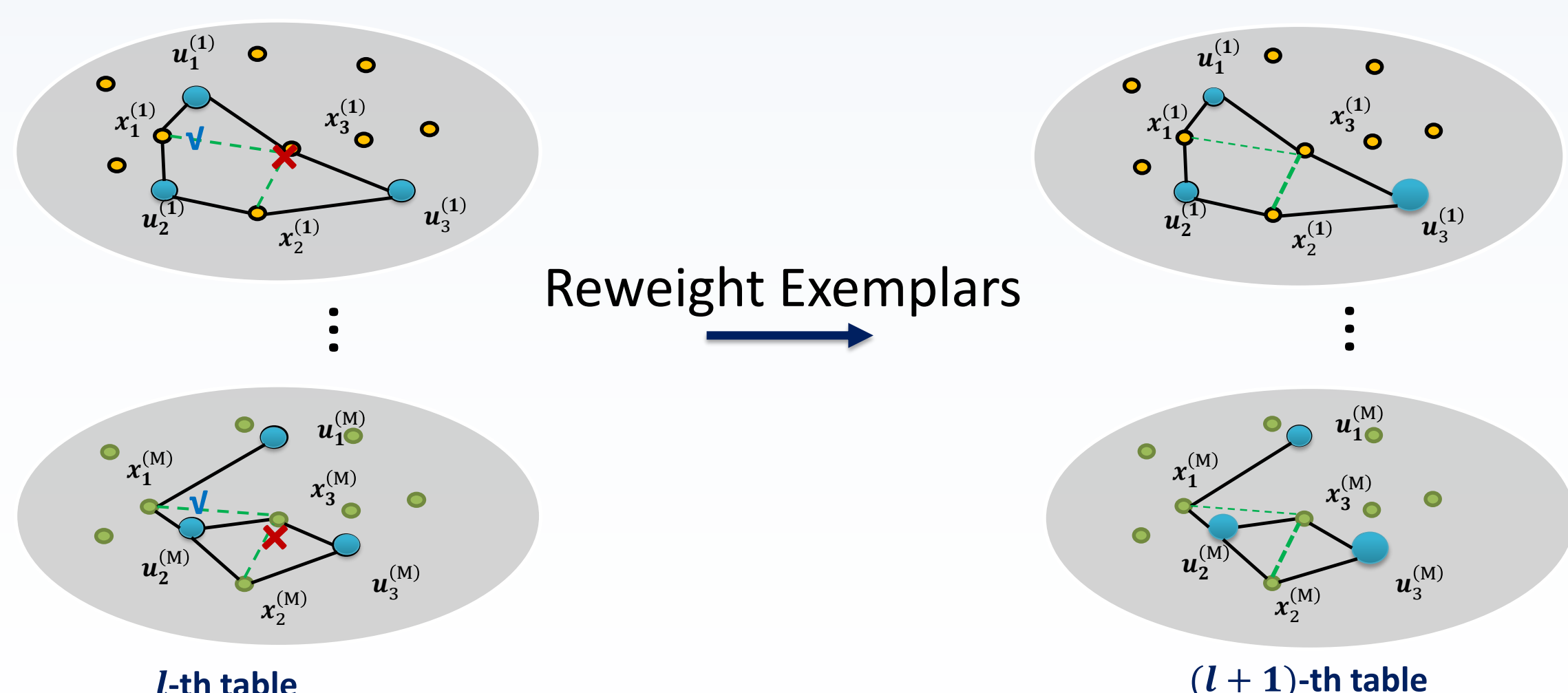
Exemplar Reweighting

- pursue the table complementarity and meanwhile preserve the low-rank similarity: calibrate the role of each exemplar

$$\hat{\mathbf{Z}}^{(m)} = \mathbf{\Gamma}^{-1} \mathbf{Z}^{(m)} \mathbf{\Pi}^{(m)}$$

Proposition 1: Theorem 1 still holds when using the nonlinear feature map based on exemplar reweighting

$$\mathbf{z}^{(m)}(\mathbf{x}) = \frac{[\delta_1^{(m)} \pi_1^{(m)} \mathcal{K}(\mathbf{x}^{(m)}, \mathbf{u}_1^{(m)}), \dots, \delta_K^{(m)} \pi_K^{(m)} \mathcal{K}(\mathbf{x}^{(m)}, \mathbf{u}_K^{(m)})]}{\sum_{k=1}^K \delta_k^{(m)} \pi_k^{(m)} \mathcal{K}(\mathbf{x}^{(m)}, \mathbf{u}_k^{(m)})}$$



✓ Correct neighbor prediction ✗ Wrong neighbor prediction

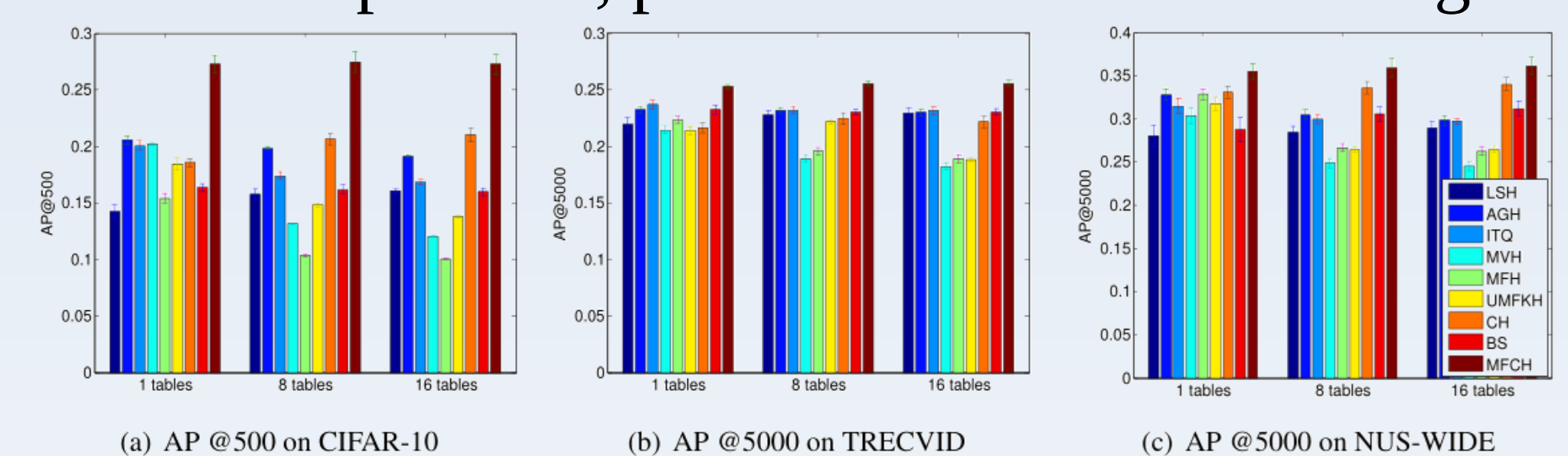
Experiments

Datasets

- CIFAR-10: 60K, 384D GIST + 300D SIFT BoW
- TRECVID: 250K, 512-D GIST + 1000-D spatial pyramid SIFT BoW
- NUS-WIDE: 270K, 128D texture + 225D color + 500-D SIFT BoW

Results

- Hash table lookup: recall, precision within a Hamming radius



- Hamming distance ranking: average precision of the top results

Table 1. Hash table lookup performance of different multi-table methods on CIFAR-10, TRECVID and NUS-WIDE.

	METHODS	RH2				PH2			
		$L=1$	$L=4$	$L=8$	$L=16$	$L=1$	$L=4$	$L=8$	$L=16$
CIFAR-10	LSH	0.71 ± 0.11	2.67 ± 0.16	5.06 ± 0.48	9.07 ± 0.25	14.67 ± 0.51	15.25 ± 0.27	15.01 ± 0.35	14.71 ± 0.19
	AGH	3.15 ± 0.17	4.53 ± 0.17	5.63 ± 0.16	7.59 ± 0.18	20.02 ± 0.32	18.28 ± 0.34	16.66 ± 0.13	14.97 ± 0.11
	ITQ	3.49 ± 0.19	5.86 ± 0.28	7.68 ± 0.20	10.81 ± 0.15	19.60 ± 0.38	16.98 ± 0.33	15.92 ± 0.28	15.04 ± 0.19
	MFH*	4.68 ± 0.49	-	-	-	22.67 ± 0.12	-	-	-
	MVH	1.58 ± 0.03	1.43 ± 0.02	2.31 ± 0.02	4.03 ± 0.02	20.54 ± 0.07	15.08 ± 0.08	12.94 ± 0.04	11.72 ± 0.03
	UMFHK	1.10 ± 0.07	1.98 ± 0.18	3.02 ± 0.07	4.50 ± 0.04	19.26 ± 0.70	15.53 ± 0.44	14.15 ± 0.17	12.76 ± 0.07
	CH	0.71 ± 0.03	3.46 ± 0.15	5.66 ± 0.16	8.73 ± 0.33	20.06 ± 0.38	19.33 ± 0.41	19.06 ± 0.37	18.47 ± 0.40
	BS	3.05 ± 0.55	5.80 ± 0.56	7.90 ± 0.57	10.68 ± 0.59	16.18 ± 0.26	15.31 ± 0.34	15.01 ± 0.32	14.44 ± 0.23
TRECVID	MVCH	5.55 ± 0.20	9.18 ± 0.28	11.19 ± 0.31	13.32 ± 0.39	26.21 ± 0.76	25.02 ± 0.75	24.38 ± 0.69	23.76 ± 0.59
	LSH	1.23 ± 0.38	4.78 ± 0.52	7.64 ± 0.40	12.25 ± 0.19	22.93 ± 0.92	22.67 ± 0.93	22.36 ± 0.88	22.06 ± 0.75
	AGH	4.75 ± 0.64	5.67 ± 0.70	6.15 ± 0.66	7.09 ± 0.59	22.97 ± 0.42	22.70 ± 0.44	22.39 ± 0.47	21.77 ± 0.48
	ITQ	4.26 ± 0.23	8.02 ± 0.61	10.90 ± 0.69	11.66 ± 0.77	23.99 ± 0.44	22.89 ± 0.55	22.22 ± 0.36	21.71 ± 0.31
	MFH	1.96 ± 0.23	3.73 ± 0.26	6.57 ± 0.52	9.93 ± 0.66	23.11 ± 0.27	20.75 ± 0.34	19.19 ± 0.35	18.42 ± 0.40
	MVH	0.73 ± 0.03	0.55 ± 0.03	0.82 ± 0.03	1.36 ± 0.03	22.77 ± 0.79	21.16 ± 0.62	19.47 ± 0.49	18.39 ± 0.43
	UMFHK	0.36 ± 0.03	0.99 ± 0.07	1.61 ± 0.12	2.00 ± 0.06	23.81 ± 0.52	22.23 ± 0.24	21.46 ± 0.19	19.44 ± 0.13
	CH	0.43 ± 0.01	3.61 ± 0.44	6.10 ± 0.45	8.92 ± 0.61	24.13 ± 0.83	22.89 ± 0.26	22.14 ± 0.26	21.45 ± 0.38
NUS-WIDE	BS	2.66 ± 0.39	5.80 ± 0.64	8.87 ± 1.26	13.02 ± 0.33	23.65 ± 0.33	22.75 ± 0.28	22.40 ± 0.30	21.99 ± 0.30
	MVCH	5.07 ± 1.20	9.09 ± 0.77	10.94 ± 0.86	12.77 ± 0.94	24.46 ± 0.62	23.86 ± 0.56	23.59 ± 0.57	23.34 ± 0.58
	LSH	0.38 ± 0.07	1.29 ± 0.08	2.32 ± 0.26	5.46 ± 0.39	30.32 ± 1.01	30.23 ± 1.15	30.15 ± 1.13	29.71 ± 1.52
	AGH	1.31 ± 0.04	1.79 ± 0.07	2.12 ± 0.07	2.82 ± 0.09	34.34 ± 0.21	32.27 ± 0.14	30.62 ± 0.11	28.82 ± 0.15
	ITQ	2.01 ± 0.30	4.51 ± 0.68	6.25 ± 0.27	8.11 ± 0.43	33.36 ± 1.36	30.05 ± 1.00	28.63 ± 1.05	28.46 ± 1.29
	MFH	0.32 ± 0.01	0.57 ± 0.01	0.87 ± 0.01	1.44 ± 0.01	34.50 ± 0.79	30.55 ± 0.53	28.14 ± 0.48	26.51 ± 0.45
	MVH	0.15 ± 0.01	0.32 ± 0.01	0.60 ± 0.01	1.15 ± 0.01	32.63 ± 1.08	26.82 ± 0.67	25.36 ± 0.58	24.57 ± 0.53
	UMFHK	0.31 ± 0.01	0.94 ± 0.18	1.35 ± 0.17	1.63 ± 0.04	36.46 ± 0.73	29.71 ± 0.28	28.03 ± 0.40	26.26 ± 0.44
	CH	0.22 ± 0.00	3.14 ± 0.03	5.18 ± 0.21	7.33 ± 0.22	36.23 ± 0.63	32.96 ± 0.52	33.12 ± 0.28	32.69 ± 0.43
	BS	0.77 ± 0.02	1.84 ± 0.01	3.01 ± 0.04	5.44 ± 0.18	33.17 ± 0.75	30.25 ± 0.02	29.48 ± 0.84	29.30 ± 0.87
	MVCH	2.17 ± 0.17	4.18 ± 1.03	6.02 ± 1.07	8.95 ± 1.15	35.67 ± 1.06	34.88 ± 1.03	34.60 ± 1.45	34.29 ± 1.59

* We didn't get a reasonable performance by tuning parameters of MFH on CIFAR-10 when using multiple hash tables. Similar results are shown in Figure 2.

Conclusion

- The First Multi-View Complementary Multi-Table Method
- Exemplar-based feature fusion: adaptively exploit multi-view information and guarantee the fast computation
- Exemplar reweighting: eliminate table redundancy in a fast boosting manner